Barker College

Student Number

Mathematics

2016

TRIAL HIGHER SCHOOL CERTIFICATE

AM Friday 5th August

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ○ (C) ○ (D) ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ☒ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct

(A) ● (B) ☒ (C) ○ (D) ○

Start Here →

1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○
5. A○ B○ C○ D○
6. A○ B○ C○ D○
7. A○ B○ C○ D○
8. A○ B○ C○ D○
9. A○ B○ C○ D○
10. A○ B○ C○ D○

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Barker College

Student Number

2016
TRIAL HIGHER SCHOOL CERTIFICATE

Mathematics

Staff Involved: **AM Friday 5th August**

- **RMH*** • **JGD***
- **DZP** • **MRB**
- **AXD** • **LAK**
- **AJD** • **KJL**
- **GPF** • **ARM**

120 copies

General Instructions

- **Reading time – 5 minutes**
- **Working time - 3 hours**
- **Write using black pen.**
- **A Reference Sheet is provided**
- **Approved calculators and Mathaids may be used.**
- **Diagrams are not to scale unless indicated.**
- **Marks may not be awarded for careless or badly arranged working.**
- **In Questions 11 – 16, show all relevant mathematical reasoning and / or calculations**

Total marks – 100

Section I **Pages 4 - 7**

10 marks
- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II **Pages 9 - 18**

90 marks
- Attempt Questions 11 – 16
- Start each question in a NEW booklet.
- Allow about 2 hours 15 minutes for this section

Section I – Multiple Choice (10 marks)
Attempt questions 1 – 10
All about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

---

1. What is 75680241 written in scientific notation, correct to 3 significant figures?

(A) $7\cdot568 \times 10^7$ (B) $7\cdot57 \times 10^7$ (C) $7\cdot568 \times 10^4$ (D) $7\cdot57 \times 10^5$

2. All students in a particular TAFE course sit a theory test and a practical test. 65% of students pass the theory test and only 40% pass the practical test. A student is chosen at random. The probability that the student passes both tests is:

(A) $0\cdot26$

(B) $1\cdot05$

(C) $0\cdot026$

(D) $2\cdot6$

3. The solution to $2^{3x+5} = 4^{x-1}$ is:

(A) $x = -2$

(B) $x = -1$

(C) $x = -6$

(D) $x = -7$

4. The parabola $y^2 = 12x - 24$ has:

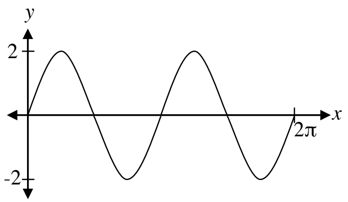
(A) focus $(5,0)$ and directrix $x = -1$

(B) focus $(2,3)$ and directrix $y = -3$

(C) focus $(-1,0)$ and directrix $x = 5$

(D) focus $(27,0)$ and directrix $x = 21$

5.



The graph above could have as its equation:

(A) $y = 2\cos\frac{x}{2}$

(B) $y = 2\cos x$

(C) $y = 2\sin x$

(D) $y = 2\sin 2x$

6. What is the value of the derivative of $y = \tan x - 3\sin 2x$ at $x = 0$?

(A) 0

(B) 7

(C) -5

(D) -2

7. The domain and range of $y^2 = 4 - x^2$ are:

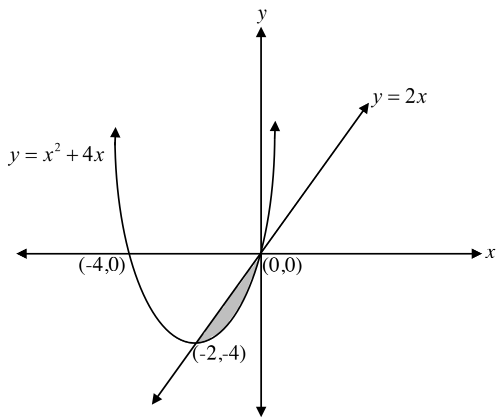
(A) $-2 \le x \le 2,\quad 0 \le y \le 2$

(B) $-2 \le x \le 2,\quad -2 \le y \le 2$

(C) all real x, $y \le 2$

(D) $x \le 4$, all real y

8. The diagram shows the parabola $y = x^2 + 4x$ meeting the line $y = 2x$ at $(-2,-4)$ and $(0,0)$.



Which expression gives the area of the shaded region bounded by the parabola and the line?

(A) $\int_{-2}^{0} \left(-2x - x^2\right)\, dx$

(B) $\int_{0}^{-2} \left(-2x - x^2\right)\, dx$

(C) $\int_{-2}^{0} \left(x^2 + 2x\right)\, dx$

(D) $\int_{0}^{-2} \left(x^2 + 2x\right)\, dx$

9. The sum of the first three terms of a geometric series is 19 and the sum to infinity is 27. The values of the first term a, and the common ratio r are:

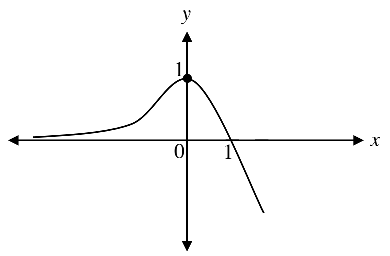
(A) $a = -9,\quad r = \frac{-2}{3}$

(B) $a = 9,\quad r = \frac{2}{3}$

(C) $a = \frac{19}{3},\quad r = \frac{19}{27}$

(D) $a = -9,\quad r = \frac{2}{3}$

10. The diagram shows the graph of $y = e^x(1-x)$.



How many solutions are there to the equation $e^x(1-x) = x^2 - 1$?

(A) 0

(B) 1

(C) 2

(D) 3

End of Section I

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Section II

90 marks
Attempt Questions 11 - 16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **[START A NEW BOOKLET]**

(a) Simplify $16a - (3-4a)$. **1**

(b) Factorise fully $8x^3 - 27$. **2**

(c) Express $\dfrac{3}{2-\sqrt{5}}$ with a rational denominator. **2**

(d) Find $\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x-3}$. **2**

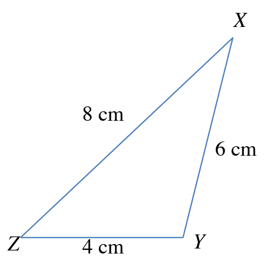
(e) Differentiate $y = \dfrac{2x}{(e^x+1)^3}$. **2**

(f) The roots of the equation $x^2-7x+9=0$ are α and β. **2**
Find the value of $\alpha^2 + \beta^2$.

(g) Differentiate $y = x^2 \cos x$. **2**

(h) The diagram shows XYZ with sides $XY = 6$ cm, $YZ = 4$ cm and $XZ = 8$ cm. **2**

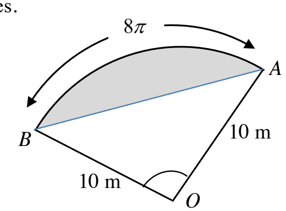
Calculate $\angle XYZ$ to the nearest degree.



End of Question 11

Question 12 (15 marks) **[START A NEW BOOKLET]**

(a) In the diagram, AB is an arc of a circle with centre O. The radius is 10 metres and the arc length is 8π metres.



(i) Show that $\angle BOA$ is $\dfrac{4\pi}{5}$ radians. **1**

(ii) Show that the area of sector AOB is 40π . **1**

(iii) Hence, or otherwise, calculate the area of the shaded segment to the nearest whole number. **1**

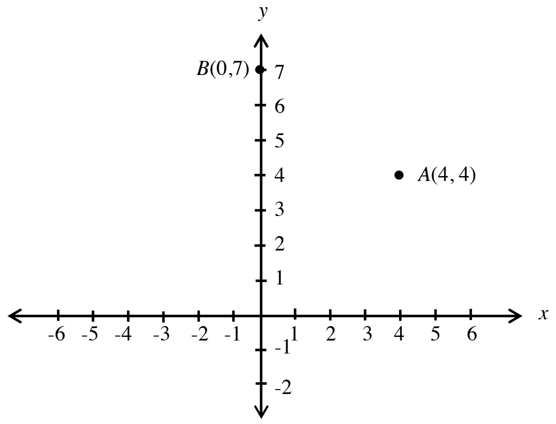
(b) Find the values of k for which the equation $x^2+kx+9=0$ has two real distinct roots. **2**

(c) Solve $2\sin^2\theta + \sin\theta = 0$ for $0 \le \theta \le 2\pi$. **3**

Question 12 continues on page 11

Question 12 (continued)

(d) The diagram below shows the points *A* (4, 4) and *B* (0, 7).



(i) Find the length of *AB*. **1**

(ii) Find the gradient of *AB*. **1**

(iii) Show that the equation of *AB* is $3x+4y-28=0$. **1**

(iv) Given the point *D*(-1 , 1), find the perpendicular distance from *D* to the line *AB*. **2**

(v) Find the coordinates of the point *C* such that *ABCD* is a parallelogram. **1**

(vi) Find the area of *ABCD*. **1**

End of Question 12

Question 13 (15 marks) **[START A NEW BOOKLET]**

(a) (i) Find the derivative of $y=3e^{x^2+1}$. **2**

(ii) Hence, or otherwise, find $\int x\, e^{x^2+1}\, dx$. **1**

(b) Find the equation of the curve $y=f(x)$ given that the curve has a turning point at $x=1$, $f'(x)=3x^2-6x+c$ and $f(2)=7$. **3**

(c) A bag contains 10 blue counters, 8 red counters and 5 green counters. If two counters are drawn from the bag and the first is not replaced, find the probability that:

(i) the second counter drawn is green, given that the first counter drawn is blue. **1**

(ii) both of the counters are blue. **1**

(iii) both counters are the same colour. **2**

(d) Jack borrows \$300 000 to buy a unit. Interest is calculated monthly at the rate of 4.8% p.a compounded monthly. He agrees to repay the loan with equal monthly instalments of \$*M* at the end of each month for 15 years. Let A_n be the amount owing after n months.

(i) Find an expression for A_1. **1**

(ii) Show that $A_{180} = 300\,000(1.004)^{180} - M\left(1+1.004+....+1.004^{179}\right)$. **2**

(iii) Calculate the amount of the monthly repayment, to the nearest cent. **2**

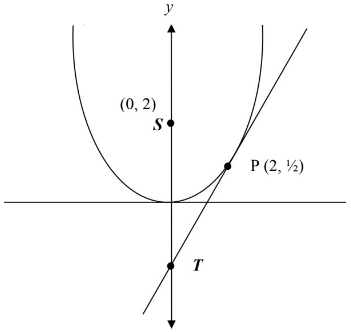
End of Question 13

Question 14 (15 marks) **[START A NEW BOOKLET]**

(a) Evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \sec^2 2x \, dx$. **3**

(b) Calculate the area bounded by the curve $y = x^2 - 7x + 10$ and the x-axis. **3**

(c) The diagram shows the parabola $x^2 = 8y$ with focus $S\ (0, 2)$. A tangent to the parabola is drawn at $P\left(2, \frac{1}{2}\right)$.



(i) Find the gradient of the tangent at point P. **1**

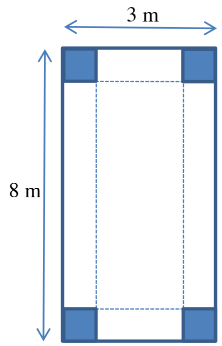
(ii) The tangent at P cuts the y-axis at T.
Find the coordinates of T. **1**

(iii) Calculate the acute angle (to the nearest minute) that the tangent at P makes with the y-axis. **2**

(iv) Calculate the area of ΔPTS . **1**

Question 14 continues on page 14

(d) A rectangular sheet of metal is 8 m by 3m.
Four equal squares, side x m, are removed from each corner.
The edges are then turned up to form a box, open at the top.



(i) Show that the volume of the box is given by
$V = 4x^3 - 22x^2 + 24x$. **1**

(ii) Find the value of x which makes this volume a maximum. **3**

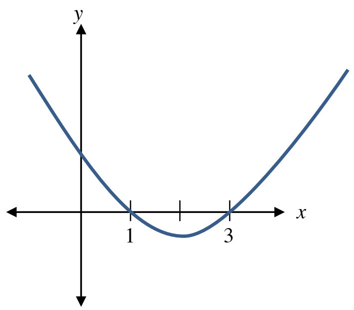
End of Question 14

Question 15 (15 marks) **[START A NEW BOOKLET]**

(a) (i) Find $\frac{dy}{dx}$ of $y = \log_e(2x-1)$. **1**

(ii) Hence, find the value of x when the gradient of the curve is $\frac{2}{5}$. **2**

(b) The graph of $y = f'(x)$ is drawn below.



Draw a possible sketch of $y = f(x)$ clearly showing the x-coordinates of any stationary points or points of inflexion. **2**

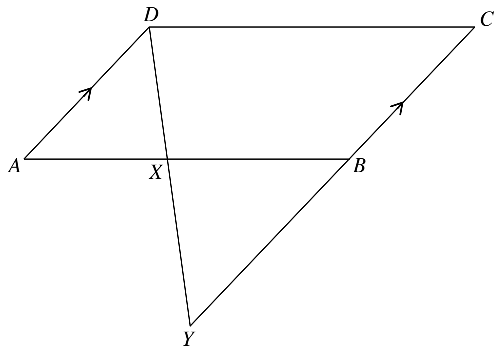
(c) Find $\int \frac{x^2}{x^3+4}\,dx$ **2**

Question 15 continues on page 16

Question 15 (continued)

(d) Use Simpson's Rule, with 5 function values, to estimate the area between the curve $y = \frac{2}{x^2-1}$ and the x-axis from $x = 2$ to $x = 6$ (answer to 3 decimal places). **3**

(e) $ABCD$ is a parallelogram.
X lies on AB.
DX and CB are both produced to Y.



(i) Prove ΔADX is similar to ΔCYD. **3**

(ii) Determine the length of XY given that $AX = 8$ cm, $DC = 12$ cm and $DX = 10$ cm. **2**

End of Question 15

Question 16 (15 marks) **[START A NEW BOOKLET]**

(a) Consider the equation $y=(x-2)^3(x+1)$.

(i) Show that $\frac{dy}{dx}=(x-2)^2(4x+1)$. **1**

(ii) Given that $\frac{d^2y}{dx^2}=6(x-2)(2x-1)$ (DO NOT PROVE THIS),

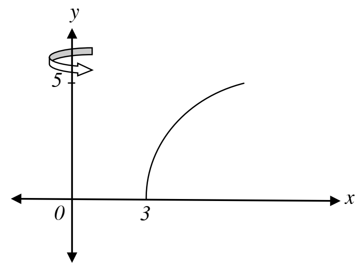
find the coordinates of any stationary points and determine their nature. **3**

(iii) Determine the coordinates of any points of inflection. **2**

(iv) Hence, sketch the curve $y=(x-2)^3(x+1)$ showing intercepts on the axes and any information from Parts (i) to (iii). **2**

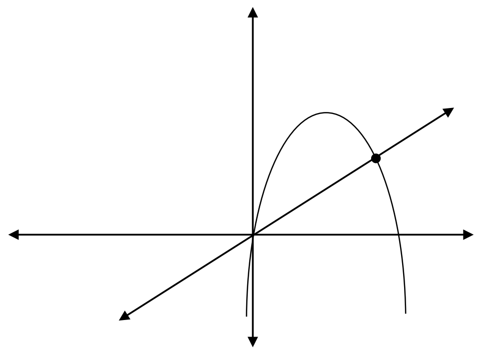
Question 16 continues on page 18

(b) A bowl is formed by rotating the curve $y=5\ \ln(x-2)$ about the y axis for $0\le y\le 5$.



Find the volume of the bowl, giving your answer as a simplified exact value. **3**

(c) The graphs of $y=mx$ and $y=6x-x^2$ intersect at the origin and at point B.



Find the area, in simplest form in terms of m, bounded by $y=mx$ and $y=6x-x^2$. **4**

End of Question 16

End of Paper

MATHEMATICS: STUDENT COPY

Yr 12 2016 TRIAL HSC SOLUTIONS:

#1 IDS

SECTION 1. MULTIPLE CHOICE

① 7.57×10^{7} (B)

② $0.65 \times 0.4 = 0.26$ (A)

③ $2^{3x+5} = 2^{2x-2}$

$\therefore 3x+5 = 2x-2$

$\therefore x = -7$ (B)

④ $y^2 = 12(x-2)$

$\therefore 4a = 12$

$a = 3 \quad V(2,0)$

$x = -1, \; S(5,0)$ (A)

⑤ $y = 2\sin 2x$ (B)

⑥ $y' = \sec^2 x - 3\sin 2x\,(2)$

At $x = 0 \quad y' = 1 - 6(0)$

$y' = -5$ (C)

⑦  (B)

⑧ $\int_{-2}^{0} 2x - (x^2 + 4x)\,dx$ (A)

$= \int_{-2}^{0} (-2x - x^2)\,dx$ or (D)

⑨ $\frac{a(1-r^3)}{1-r} = 19$

$\frac{a}{1-r} = 27$

$\therefore 27(1-r^3) = 19$

$27 - 27r^3 = 19$

$\therefore -27r^3 = -8$

$r^3 = 8/27$

$r = 2/3$

So $r = 2/3 \;\therefore \frac{a}{1-r} = 27$

$\therefore \quad a = 9$ (B)

⑩ 

Ans: 2 (C)

Section II next page.

SECTION II Solutions 2016 Mathematics Trial

Question 11.

(a) $16a - 3 + 4a = 20a - 3$

(b) $(2x)^3 - (3)^3 = (2x-3)(4x^2+6x+9)$

(c) $\frac{3}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{6+3\sqrt{5}}{4-5}$

$= -6 - 3\sqrt{5}$

(d) $\lim_{x \to 3} \frac{(x-3)(x+5)}{(x-3)} = 3+5$

$= 8$

(e) $y' = \frac{(e^x+1)^3 \cdot (2) - 2x \cdot 3(e^x+1)^2 \cdot e^x}{(e^x+1)^6}$

$= \frac{2(e^x+1)^3 - 6x e^x (e^x+1)^2}{(e^x+1)^6}$

$= \frac{(e^x+1)^2 \left[2e^x + 2 - 6x e^x\right]}{(e^x+1)^6}$

$= \frac{2 + 2e^x - 6xe^x}{(e^x+1)^4}$

(f) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\alpha+\beta = \frac{7}{1}$ $\quad = (7)^2 - 2 \times 9$

$\alpha\beta = \frac{9}{1}$ $\quad = 31$

(g) $y' = 2x\cos x - x^2\sin x$

(h) $\cos X\hat{Y}Z = \frac{4^2 + 6^2 - 8^2}{2 \times 4 \times 6}$

$\therefore \cos X\hat{Y}Z = -0.25$

$\therefore X\hat{Y}Z = 104°$ (to nearest degree)

Question 12.

(a) Let $B\hat{O}A = \theta$; $l = r\theta$

(i) $8\pi = 10 \times \theta$

$\therefore \theta = \frac{8\pi}{10} = \frac{4\pi}{5}$ radians

(ii) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \cdot (10)^2 \cdot \frac{4\pi}{5}$

$= 10 \times 4\pi = 40\pi \text{ m}^2$

(iii) $A = 40\pi - \frac{1}{2}(10)(10)(\sin\frac{4\pi}{5})$

$= 40\pi - 50\sin\frac{4\pi}{5}$

$\approx 96\text{m}^2$

(b) $\Delta > 0 \qquad \Delta = b^2 - 4ac$

$\therefore k^2 - 4 \times 1 \times 9 > 0$

$k^2 - 36 > 0$

$(k-6)(k+6) > 0$

$\therefore k < -6$ and $k > 6$

(c) $2\sin^2\theta + \sin\theta = 0$

$\therefore \sin\theta(2\sin\theta + 1) = 0$

$\therefore \sin\theta = 0 \qquad 2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

$\therefore \theta = 0°, \pi, 2\pi; \frac{7\pi}{6}, \frac{11\pi}{6}$

(d) (i) $AB = \sqrt{(4-0)^2 + (4-7)^2}$

$= \sqrt{16+9}$

$= 5$

Q.12 (B) cont'd.

(ii) $m_{AB} = \frac{7-4}{0-4} = \frac{-3}{4}$

(iii) $y - y_1 = m(x - x_1)$ using (0,7)

$y - 7 = \frac{-3}{4}(x - 0)$

$4y - 28 = -3x$

$\therefore 3x + 4y - 28 = 0$

(iv) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$d = \left| \frac{3\times(-1) + 4\times(1) + (-28)}{\sqrt{(3)^2 + (4)^2}} \right|$

$d = \frac{|-27|}{5} = \frac{27}{5}\,u$

(v) $C(-5, 4)$

(vi) $A = b \times h$

$= 5 \times \frac{27}{5} = 27\,u^2$

Question 13:

(a) (i) $y' = 3 \times 2x\, e^{x^2+1}$

$= 6x\, e^{x^2+1}$

Q13(a) (ii)

$\int x e^{x^2+1}\,dx = \frac{1}{6}\int 6x e^{x^2+1}\,dx$

$= \frac{1}{6} \cdot 3e^{x^2+1} + c$

$= \frac{1}{2} e^{x^2+1} + c$

(b) $f'(x) = 3x^2 - 6x + c$

$f'(1) = 0$

$\therefore f'(1) = 3 - 6 + c = 0$

$\therefore c = 3$

$\therefore f'(x) = 3x^2 - 6x + 3$

$f(x) = x^3 - 3x^2 + 3x + c$

$f(2) = 7$

$\therefore (2)^3 - 3(2)^2 + 3(2) + c = 7$

$8 - 12 + 6 + c = 7$

$c = 5$

$\therefore f(x) = x^3 - 3x^2 + 3x + 5$

(c) | 10B 8R 5G | = 23

(i) $P(G) = \frac{5}{22}$

(ii) $P(BB) = \frac{10}{23} \times \frac{9}{22}$

$= \frac{45}{253}$

(iii) $P(BB \text{ or } RR \text{ or } GG)$

$= \frac{45}{253} + \left(\frac{8}{23} \times \frac{7}{22}\right) + \left(\frac{5}{23} \times \frac{4}{22}\right)$

$= \frac{86}{506}$

$= \frac{43}{253}$

(Q13 d next page)

Q.13 cont'd

(d) Note:

\$300 000

4.8% pa ⇒ 0.4% per month

15 yrs ⇒ 180 months

(i) $A_1 = 300\,000(1.004) - M$

(ii) $A_2 = A_1 \times (1.004) - M$

$= (300000 \times 1.004 - M) \times 1.004 - M$

$= 300000 \times 1.004^2 - M(1.004) - M$

$\therefore A_3 = 300\,000(1.004)^3 - (1.004)^2 M - (1.004)M - M$

$A_3 = 300\,000(1.004)^3 - M(1 + 1.004 + 1.004^2)$

$\therefore A_{180} = 300\,000(1.004)^{180} - M(1 + 1.004 + 1.004^2 \ldots + 1.004^{179})$

(iii) When $A_{180} = 0$ loan repaid

$\therefore 300\,000(1.004)^{180} - M \times \frac{1[(1.004)^{180} - 1]}{1.004 - 1} = 0$

$\therefore M = \frac{300\,000(1.004)^{180} \times 0.004}{(1.004^{180} - 1)}$

$M = \$2341.24$

Question 14:

(a) $\int_{\pi/8}^{\pi/6} \sec^2 2x\,dx$

$= \frac{1}{2}\left[\tan 2x\right]_{\pi/8}^{\pi/6}$

$= \frac{1}{2}\left[\tan \frac{\pi}{3} - \tan \frac{\pi}{4}\right]$

$= \frac{1}{2}\left[\sqrt{3} - 1\right]$

(b) Now $x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

$\therefore x = 2, 5$

$A = \left| \int_2^5 (x^2 - 7x + 10)\,dx \right|$

$= \left| \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x \right]_2^5 \right|$

$= \left| \left(\frac{5^3}{3} - \frac{7\times 5^2}{2} + (5\times 10)\right) - \left(\frac{2^3}{3} - \frac{7\times 2^2}{2} + 20\right) \right|$

$= \left| \frac{25}{6} - \frac{26}{3} \right|$

$= \left| -\frac{9}{2} \right| = \frac{9}{2}\,u^2$

(c) $x^2 = 8y \quad \therefore y = \frac{x^2}{8}$

(i) $y = \frac{x^2}{8}$

$y' = \frac{x}{4}$

at $x = 2 \quad \therefore m = \frac{2}{4} = \frac{1}{2}$

(ii) Next column. next page

Q 14: cont'd

(c) (ii) $y - \frac{1}{2} = \frac{1}{2}(x - 2)$

$2y - 1 = x - 2$

When $x = 0$, $2y - 1 = -2$

$y = -\frac{1}{2}$

$\therefore T(0, -\frac{1}{2})$

(iii) If $m = \tan\theta$

$\therefore \tan\theta = \frac{1}{2}$

$\therefore \theta = \tan^{-1}(\frac{1}{2})$

$\theta = 26°34'$

$\therefore$ Angle with y-axis

$90° - 26°34' = \underline{63°26'}$

(iv)

$A = \frac{1}{2} \times 2 \times 2.5$

$= 2.5\ u^2$

(d)

(i) $V = L \times b \times h = (3-2x)(8-2x)(x)$

$= (24 - 6x - 16x + 4x^2)x$

$= 24x - 22x^2 + 4x^3$

Q14 d (ii) $V = 24x - 22x^2 + 4x^3$

$\frac{dV}{dx} = 24 - 44x + 12x^2$

Let $\frac{dV}{dx} = 0$

$\therefore 24 - 44x + 12x^2 = 0$

$4(6 - 11x + 3x^2) = 0$

$4(3 - x)(2 - 3x) = 0$

$\therefore x = 3, \frac{2}{3}$

Now $\frac{d^2V}{dx^2} = -44 + 24x$

When $x = 3$ $\frac{d^2V}{dx^2} = -44 + 24 \times 3$

$= 28$

$\therefore \frac{d^2V}{dx^2} > 0$ Min

When $x = \frac{2}{3}$ $\frac{d^2V}{dx^2} = -44 + 24 \times \frac{2}{3}$

$\frac{d^2V}{dx^2} = -28$

$\therefore \frac{d^2V}{dx^2} < 0$ Max

There maximum volume is when $x = \frac{2}{3}$

(No need to state the volume).

P.T.O.

Question 15:

(a)(i) $y = \log_e(2x-1)$

$y' = \frac{2}{2x-1}$

(ii) $\frac{2}{2x-1} = \frac{2}{5}$

$\therefore 2x - 1 = 5$

$2x = 6$

$x = 3$

(b)

(c) $\frac{d}{dx}(x^3 + 4) = 3x^2$

$\therefore \frac{1}{3}\int \frac{3x^2}{x^3+4}\,dx$

$= \frac{1}{3}\ln(x^3+4) + C$

(d)

x	2	3	4	5	6
$f(x)$	2/3	1/4	2/15	1/12	2/35

$A \doteq \frac{1}{3}\left[\left(\frac{2}{3} + \frac{2}{35}\right) + 4\left(\frac{1}{4} + \frac{1}{12}\right) + 2\left(\frac{2}{15}\right)\right]$

$= \frac{1}{3} \times \frac{241}{105}$

$= \frac{241}{315}\ u^2$

Q15 cont'd

(e)

(i) In $\triangle ADX$ and $\triangle CYD$

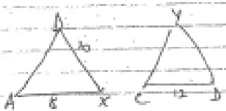
$A\hat{D}X = X\hat{Y}C$ (alt. ∠s in ∥ lines are equal)

$A\hat{X}D = Y\hat{X}B$ (vertically opp.)

$\therefore D\hat{A}X = X\hat{B}Y$ (angle sum of △)

$\therefore \triangle ADX \,|||\, \triangle CYD$ (equiangular)

(ii)



$\frac{YD}{10} = \frac{12}{8}$ (sides similar △s in same ratio)

$\therefore YD = \frac{120}{8} = 15$

$\therefore XY = 15 - 10$

$= 5\text{cm}$

Section II: Year 12 Maths Trial Solution 2016

Question 16:

$y = (x-2)^3(x+1)$

(a)

(i) $\frac{dy}{dx} = (x-2)^3 \cdot 1 + (x+1)3(x-2)^2 \cdot 1$

$= (x-2)^2\left[(x-2) + 3(x+1)\right]$

$= (x-2)^2(4x+1)$

(ii) $\frac{d^2y}{dx^2} = 6(x-2)(2x-1)$

$\therefore \frac{dy}{dx} = 0$ stat pts

$\therefore (x-2)^2(4x+1) = 0$

$\therefore x = 2, -1/4$

Now

When $x=2$ $\frac{d^2y}{dx^2} = 0$, possible p.o.I.

$x = -1/4$ $\frac{d^2y}{dx^2} = 6(-9/4)(-3/2)$

$= \frac{+81}{4}$

$\therefore \frac{d^2y}{dx^2} > 0$

Min $\left(-1/4, \frac{-2187}{256}\right)$

Now At $x = 2$

x	1	2	3
y'	-6	0	30

Changes sign

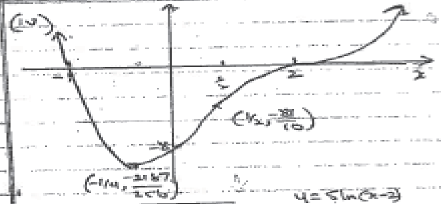
horizontal pt of inflexion at $(2, 1/2)$.

(iii) Let $d^2y/dx^2 = 0$ at $x = 2, 1/2$

x	0	1/2	1
y''	12	0	-6

point of inflexion $(1/2, -81/16)$

(iv)



(b) $V = \pi \int_0^5 x^2\, dy$

$y = 5\ln(x-2)$

$\frac{y}{5} = \ln(x-2)$

$\therefore e^{y/5} = x-2$

$x = 2 + e^{y/5}$

$V = \pi \int_0^5 (2 + e^{y/5})^2\, dy$

$= \pi \int_0^5 (4 + 4e^{y/5} + e^{2y/5})$

$= \pi\left[4y + \frac{4e^{y/5}}{1/5} + \frac{e^{2y/5}}{2/5}\right]_0^5$

$= \pi\left[4y + 20e^{y/5} + 5/2\, e^{2y/5}\right]_0^5$

$= \pi\left[\left(20 + 20e + \frac{5e^2}{2}\right) - \left(0 + 20 + 5/2\right)\right]$

$= \pi\left[5e^2/2 + 20e - 5/2\right] u^3$

(c) (Pt. B) $mx = 6x - x^2$

$\therefore x^2 + mx - 6x = 0$

$x(x + (m-6)) = 0$

$\therefore x = 0 \ ; \ 6-m$

So $A = \int_0^{6-m} 6x - x^2 - mx\, dx$

$= \left[3x^2 - x^3/3 - \frac{mx^2}{2}\right]_0^{6-m}$

$= \left[\left(3(6-m)^2 - \frac{(6-m)^3}{3} - \frac{m}{2}(6-m)^2\right)\right] - 0$

$= (6-m)^2\left[3 - \frac{(6-m)}{3} - \frac{m}{2}\right]$

$= (6-m)^2\left[18 - 2(6-m) - 3m\right]$

$= \frac{(6-m)^2}{6} \cdot \frac{(6-m)}{1} = \frac{(6-m)^3}{6}$

END